Assignment 0

Date Due: Thursday, Jan. 21

Reading: Chapter 1, pages 1-7
Chapter 3, pages 25-40
Chapter 5, pages 63-75
Review Chapter 2 and Chapter 4, pages 8-20 and 47-59

This homework will not be graded.

Please Note: We will only use Chapters 2 and 4 as needed. You should look through these two chapters, reading parts of chapters 2 and 4 which are unfamiliar.

Problems:

1. Prove by induction that:
   For all integers $n \geq 25$, $5^n \geq n^{10}$

2. Find the error in the following proof by induction.

   Claim: In any set of $h$ horses, $h \geq 1$, all horses in the set are the same color.

   Proof: By induction on $h$.

   Base Case: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.

   Inductive Step: For $k > 0$ assume the claim is true for $h = k$ and prove that it is true for $h = k + 1$.

   Take any set $H$ of $k + 1$ horses. we will show that all horses in this set are the same color. Remove one horse from this set to obtain the set $H_1$ with just $k$ horses. By the induction hypothesis, all the horses in $H_1$ are the same color. Now replace the removed horse and remove a different one to obtain the set $H_2$. By the same argument, all the horses in $H_2$ are the same color. Therefore, all the horses in $H$ must be the same color and the proof is complete. (Exercise due to Mike Sipser)

3. Let $R$ be the relation on $N$ defined by, $R = \{(x, y) \mid x + y \in N$ and $x + y$ is even $\}$

   (i) Prove that $R$ is an equivalence relation.

   (ii) How many equivalence classes are there for this relation and what are they?
4. For each of the following 3 statements about $\mathbb{Z}$, state if they are true or false and say why.

a. $\forall x, y \exists z \ (x + z = y)$

b. $\forall x, y \exists z \ (x \cdot z = y)$

c. $\exists n \forall m \exists t \ (nm + mt > 0)$

5. Write the negation of the statements in problem 4.