Assignment 1

Date Due: Thursday, Jan. 28 at 5:00

Reading: Chapters 4 and 5, pages 53-92
In the second edition the reading is: Chapter 4, pages 47-59
Chapter 6, pages 76-90

Problems:

1. Prove that, if \(a \mid b\) and \(a \mid c\) then \(a \mid (b,c)\).

2. True or False? For any \(a,b,c\) in \(\mathbb{N}\), if \(a \mid bc\) then \(a \mid b\) or \(a \mid c\).
   Justify your answer.

3. Use induction to prove that for all \(n > 0\), \(1 + 2^1 + 2^2 + \ldots + 2^n = 2^{n+1} - 1\).

4. Show that if \(a \mid b\) then \((a,b) = a\).

5. Problem 17, on page 59. (In Ed. 2 it is Page 52, problem E7.)

6. Problem 18, on page 59. (In Ed. 2 it is Page 52, problem E8.)

7. Prove that there are three positive integers \(a, b,\) and \(c\), with \((a,b) > 1\), and \((b,c) > 1\) and \((a,c) > 1\)
   but \((a,b,c) = 1\).

8. Problem 24, on page 59. (In Ed. 2 it is Page 52, problem E13.)