Date Due: Thursday, April 29 at 5:00

Reading: Chapter 12, page 271-274, chapter 13, pages 285-292, and chapter 15, pages 307-318

Problems:


2. Find a positive integer a which is less than 140 and such that \( a \equiv 5^{288} \mod 140 \).

3. Page 205, problem 4
   Note: there is a small error in this problem, as \( e=4 \) will not work. Please use \( e = 5 \) instead.

4. Page 205, problem 7

5. Page 261, problem 3

6. Use the Chinese remainder theorem to find the smallest non-negative solution to \( x=7 \mod 13 \) and \( x = 4 \mod 15 \)

7. John Smith is a bad student. He tried to use Chinese Remainder theorem with the modulo the values \( a = 12, b = 14 \), even though he knows perfectly well that we only proved the Chinese Remainder Theorem for relatively prime values. Show him the error of his ways by giving a pair of values that have (at least) two Chinese Remainders that are the same even though they aren’t the same modulo \( 12 \times 14 = 168 \).

To be precise, I am asking you to find two integers, say \( v \) and \( w \), such that \( x = v \mod 12 \) and \( x = w \mod 14 \) have two different solution for \( x \), and these two different solution are not equal mod 168.

8. Compute \( (x^3 + x^2 + 1)(x^4 + 2x^2 + 2) \) in \( \mathbb{Z}_3[x] \)

9. Divide \( x^3 + x^2 - 5x - 3 \) by \( (x-2) \) in \( \mathbb{Q}[x] \).

10. Page 306, problem 50. Show your work, that is show that you get a quotient and remainder that are not unique.