We are given sets $A = \{x,y,z\}$, $B = \{y,z,w\}$, $C = \{y,w\}$. True or false?

1. $(A \cap B) \cup C = C$

2. $(A \cap C) \cup B \subseteq C$

3. $A - B = A - C$

In this class we will talk about alphabets $A$ and languages $L$. An alphabets $A$ is any non-empty finite set. We let $A^*$ be the set of all finite strings of elements from $A$. So $A^*$ is infinite. Languages are sets whose elements come from some $A^*$, so they are subsets of $A^*$.

Consider the language $L = \{x \mid x$ is a binary string starting with 11 and ending with a 1 followed by an even number of 0’s $\} = \{x \mid x = 11y10^t \text{ where } y \in \{0,1\}^* \text{ and } t \geq 2 \text{ is even } \}$.

4. What is the alphabet of $L$? (This answer is not unique.)

5. Give an example of a subset $B$ of $L$ with $|B| = 5$.

6. Give an example of a subset $B$ of $L$ which is infinite and for which $L - B$ is also infinite.

7. Give an example of a language $K$ with the same alphabet as $L$ and such that $K-L = \{0111, 100, 11, 000, 1010000\}$.

8. True or false: Any two languages are equal if and only if they have the same elements.
For any set $H$, define the power set of $H = \mathcal{P}(H) = \text{the set of all subsets of } H$. And define the size of $H = |H| = \text{the number of elements in } H$.

9. and 10. Let $D = \{ a, 2, B \}$, so $|D| = 3$. What is $\mathcal{P}(D)$? What is $|\mathcal{P}(D)|$?

11. True or false: Any two sets are equal if they have the same subsets. (More precisely, for any two sets $A$ and $B$, if $\mathcal{P}(A) = \mathcal{P}(B)$ then $A = B$.)

12. Give an example of a function with rate of growth strictly less than $O(\log n)$.

13. Give an example of a function with rate of growth strictly between $O(2^{\sqrt{n}})$ and $O(2^n)$.

For each of the following 4 statements, say if they are true or false about the natural numbers $\mathbb{N} = \{0,1,2,3,...\}$?

14. $\forall b \exists x \ ( x < b \land x \text{ is even}).$

15. $\forall x \exists b \exists k \ ( b \geq x \land b \leq 2x \land b = 2^k)$.

16. The twin primes conjecture states that there are infinitely many natural numbers $n$ such that both $n$ and $n+2$ are prime.

Write a statement (as in problems 14 and 15) which formally states the twin primes conjecture.

17. Write the negation of statement in problem 15 above?

Do not just put a negation before the statement. Write out the negation using quantifiers and negating or changing as needed within the expression.

A correspondence is a 1-1 and onto function.

18. Write down a correspondence between $\{2,4,6,8,10\}$ and the integers strictly between -50 and -56.

19. Write down a correspondence between the positive natural numbers $\mathbb{N}^+ = \{1,2,3,...\}$ and $\{0^k1^l|k\text{ and } l \text{ are odd positive integers}\} = \text{the collection of all finite strings } s \text{ of } 0\text{'s and } 1\text{'s where } s \text{ is an odd number of } 0\text{'s followed by an odd number of } 1\text{'s. For this problem you can be a bit informal and indicate the correspondence } f \text{ by ordering the binary string in the set and letting } f(1) = \text{the first element in the ordered set, } f(2) = \text{the } 2^{nd} \text{ element in the ordered set, etc.}$

Answer true or false to the last 6 questions. Here $I = \text{the set of all (positive and negative integers)}$:

20. The function $f: I \rightarrow I$ defined by $f(x) = 2x$ is 1-1.

21. The function $f: I \rightarrow I$ defined by $f(x) = x^2$ is 1-1.

22. The function $f: I \rightarrow I$ defined by $f(x) = x^2$ is onto.

23. The function $f: I \rightarrow I$ defined by $f(x) = 2x + 3$ is onto.

24. The function $f: I \rightarrow I$ defined by $f(x) = \frac{1}{2}x$ is onto.

25. The function $f$ which maps any set $S$ to $\mathcal{P}(S)$ is 1-1.