At the top of your homework paper write your name, the class number (CS 332) and also the homework assignment number (HW 3), and write the number of the problem at the top of your problem solution for each problem.

Reading: Chapter 4, section 4.2, pages 201-210

PROBLEMS:

1. Write out the full program of a Turing machine which decides the language \( L \) of all binary strings whose corresponding integer is one more than an even power of two.
   (That is the integers 2, 3, 5, 9, 17, ...)
   (Note: The string may have some leading 0’s before the 1.)
   You should show how the computation of the TM proceeds on an example input string, and mention the purpose/actions of the various TM states you use.

2. Describe the computation of a TM which accepts the language
   \( L = \{ b \mid b \text{ is a binary string which has exactly one more 1 than 0} \} \).
   Here you can informally describe how the TM works. You need not give the full program or diagram.

3. Assume that \( S \) is a decidable set of natural numbers. Let \( S_b \) be the set of all binary strings which consists of the elements in \( S \) represented in base 2.
   i. Give an algorithm to decide \( S_b \). You may invoke Turing’s thesis here.
   ii. Is this same statement true when \( S \) is recognizable (but maybe not decidable) ?
   That is, Is it true that for any recognizable set \( S \) of natural numbers, the corresponding set \( S_b \) of binary integers is also recognizable? Explain your reasoning.

4. Show that the decidable sets are closed under intersection. That is,
   Specifically that means, show that if \( J \) and \( L \) are decided by TM’s \( M_1 \) and \( M_2 \), then there is a TM \( M_3 \) that decides \( J \cap L \).
   Suggestion: For problems 4 and 5 you might have a look at the answers to these same problems for union instead of intersection in the textbook. These are problems 3.15a and 3.16a.
5. Show that the acceptable sets are closed under intersection.

Specifically show that if J and L are accepted by TM’s M1 and M2, there is a TM M3 that accepts J \cap L.

6. Which two of the following 3 languages $L_i$ are acceptable.

For each of these languages tell how a TM that accepts them would work.

If $L_i$ is decidable given an algorithm that decides it.

$L_1 = \{ < M > \mid \text{M is a TM which accepts less than 3 strings} \}$
$L_2 = \{ < M > \mid \text{M is a TM which accepts more than 3 strings} \}$
$L_3 = \{ < M > \mid \text{M on input 0101 never moves its head past the 2nd 1 in the input} \}$

Note: It is also true that 2 of the 3 languages are undecidable. Since we don’t yet know how to prove undecidability, you need not prove any of the languages are undecidable.