CS 332 - Homework 3

Due: Thursday, February 22

Reading: Chapter 4.2 in the Sipser textbook, pages 201 - 210.
If you get a chance take a look at the articles about computation that are mentioned as extra reading on the course homepage.

PROBLEMS

Part 1: These 4 problems are to be turned in and each will be graded (10 points each).

1. (i). (8 points) Show that a set of natural numbers is decidable if and only if it can be enumerated in increasing order. (You need not consider the empty set here.)
   (ii). (2 points) Is every set of natural numbers that can be enumerated in non-increasing order decidable? Why or why not? (This question has a simple answer but is somewhat tricky.)

2. Say whether each of i. - iv. is True or False. In each case briefly justify your answer.
   i. If the Church/Turing thesis is true then any language which is recognized by a Turing machine can be decided by a TM.
   ii. There are infinitely many different decidable languages.
   iii. There is a language L which can be decided by a 3 tape TM but not recognized by any 1 tape TM.
   iv. Every Turing machine decides exactly one language.

3. Let D be some decidable language which is a subset of \( N \times N \).
   Consider the language \( S = \{ x \mid \exists y (x, y) \in D \} \).
   Prove that S is recognizable.

4. Assume M is a TM whose program only allows the tape head to move right or stay stationary. (So the tape head on its tape never moves left.)
   Prove that the language of M is decidable. In particular, give an algorithm which shows that for any input w to M we can decide if M(w) loop or halts. Use this to give an algorithm which decides \( \text{L(M)} \) = the set of strings w that M accepts.
   (Hint: To get started think about how many steps the TM can make while staying on the same tape square without repeating the same state and the same symbol on the tape square. It should be clear that if this repeated state and symbol occurs while M is on the same tape square then M is in a loop.)

Part 2: These problems are good practice and you should try them. They will not be graded
1. Consider the following nondeterministic Turing machine $N$ where $N$ has states $q_i$ for $i = 0, 1, 2, 3, 4, a, r$. $N$’s input alphabet is \{0, 1\}, $N$’s tape alphabet is \{0, 1, B\}, $N$’s has the following function $\delta$ as it’s program. Note: If the definition of $\delta$ is not specified for some pair (state, symbol) then you may assume the program simply halts and goes into the $q_r$ state when it does. So for example if at some point the TM is in state $q_2$ and reads a 0 then it goes to state $q_r$ and halts.

So here is $\delta$: $\delta(q_0, 0) = \{q_0, 0, R\}$ $\delta(q_0, 1) = \{(q_0, 1, R), (q_1, 1, R)\}$ $\delta(q_1, 0) = \{(q_0, 0, R)\}$ $\delta(q_1, 1) = \{(q_2, 1, R), (q_3, 1, R)\}$ $\delta(q_2, 1) = \{(q_a, 1, R)\}$ $\delta(q_3, 0) = \{(q_4, 0, R)\}$ $\delta(q_4, 1) = \{(q_a, 1, R)\}$

i. Run the TM on the input strings 001101 and on 1001. Does $N$ accept either of these strings? Explain why.

ii. Describe which strings are in the language $L(N)$, where $L(N) = \{w | \text{the machine } N \text{ accepts } w\}$?

2. Show that the collection of all decidable languages is closed under intersection.

3. Exercise 3.11 on page 189.

4. Exercise 3.22 on page 190.

5. Exercise 4.5 on page 211.

6. Exercise 4.8 on page 211.

7. Given a recognizable language $R \subseteq N$, show that there is some decidable language $D$ such that $R = \{x | \exists y (x, y) \in D\}$. 