CS 332 - Homework 4

Due: Thursday, March 28

At the top of your homework paper write your name, the class number (CS 332) and also the homework assignment number (HW 4), and write the number of the problem at the top of your problem solution for each problem.

Reading: Chapter 5: Section 5.1, only pages 215-218, and section 5.3, pages 234-238.
Chapter 7: Pages 275-290.

PROBLEMS:

1. i. Give an example of two undecidable languages whose union is decidable.
   
   Hint: Consider using HALT or $A_{TM}$ as one of the sets.

ii. Give an example of two undecidable languages whose intersection is infinite and decidable.

2. We say a function $f: \mathbb{N} \rightarrow \mathbb{N}$ is total computable if there is TM $M$ which start with a natural number $n$ on its tape and which halts with $f(n)$ on its tape.
   
   For example the function $f(n) = n+1$ is total computable.
   
   If we allow the TM $M$ to loop on some inputs the function is called partial computable.
   
   Explain why the range of any total computable function is a recognizable language.

3. Recall that the language $HALT_{TM} = \{<M,x>|M$ is a TM and $x$ is an input to $M$ and $M(x)$ halts $\}$ $HALT_{TM}$ was shown in class to be undecidable. (Also on page 216, Theorem 5.1 in the Sipser book.)
   
   i. Is $HALT_{TM}$ a finite language or an infinite language ?
   
   ii. Let $S \subseteq HALT_{TM}$. What do the elements of $S$ look like ?

   e.g. “An element of $S$ is a pair _______ such that _______."

   iii. Define a subset $S$ of $HALT_{TM}$ which is infinite and decidable.
   
   Note: You need to say/define here which elements of $HALT$ are in your set $S$, not just that such an $S$ exists.

4. Prove that $\{ <M>|M$ accepts the string $10 \}$ is undecidable

5. Prove that $\{ <M>|M$ never enters the state $q_4$ on any input string \}$ is undecidable.