CS 332 - Homework 4 - First Draft

Due: Thursday, March 29

Reading: Chapter 5.1 and 5.3 in the Sipser textbook, pages 213 - 216 and 234-238. So skip section 5.2 and also those results in 5.1 which refer back to Section 1 of the book. Chapter 7, pages 275 - 298.

PROBLEMS

Part 1: These 5 problems are to be turned in and each will be graded (10 points each).

1. (i). TBD

2. Say whether each of i. - iv. is True or False. In each case briefly justify your answer.
   i. There are infinitely many undecidable problems.
   ii. If $L$ is decidable and $J$ is undecidable than $L \cup J$ is undecidable.
   iii. If $L$ is decidable and $J$ is undecidable than $L \cap J$ is undecidable.
   iv. If $J$ is reducible to $K$ and $K$ is recognizable then $J$ is also recognizable.

3. Prove that the problem of deciding if a TM $M$ ever writes the symbol 1 on its tape is undecidable.
   You should do these problems using reductions.

4. Prove that it is undecidable to determine if two TM’s $M$ and $L$ accept the same languages.
   You should do these problems using reductions.

5. Assume $M$ is a TM which is restricted to always working on the non-blank part of its work tape (So the tape head on its tape never moves left off its tape or right past the last input symbol.)
   Prove that the language of $M$ is decidable. In particular, give an algorithm which shows that for any input $w$ to $M$ we can decide if $M(w)$ loop or halts. Use this to give an algorithm which decides $L(M) = \{w \mid M \text{ accepts } w\}$.
   (Hint: To get started think about how many steps the TM can make while staying on the same tape square without repeating the same state and the same symbol on the tape square.)

Part 2: These problems are good practice and you should try them. They will not be graded.
1. Note: Some of the ideas for this problem were covered in section. You can see the exact definitions you need on page 271 of our textbook.
   a. Give an example of a Boolean formula $F$ which contains at least 2 different variables, and where $F$ is satisfiable and its negation $(\neg F)$ is not satisfiable.
   b. Give an example of a Boolean formula $F$ which contains at least 3 different variables, and where $F$ is satisfiable and has exactly 4 satisfying truth assignments.

2. Page 239, problem 5.7

3. Page 239, problem 5.8