CS 332 - Homework 5

Due: Tuesday, April 17

Reading:
Chapter 7, pages 299-319, Chapter 8, pages 331-351.

PROBLEMS
Part 1: These 5 problems are to be turned in and each will be graded (10 points each).

1. Part of Problem 7.31 on page 326 of the textbook.
   In problem 7.31 the scheduling problem is defined and the book asks you to show the problem is NP-complete. You need not do this. For this homework problem just show that the scheduling problem is in NP.
   You should use a verifier to do this. Say what the input to the verifier is, what the verifier does on its input, and why the verifier runs in polynomial time.

2. The bin packing problem is: You are given a supply of k bins, each bin is of size 1, and you also are given m objects, \( o_1, o_2, \ldots, o_m \), where \( o_i \) has size \( s_i \) and \( 0 < s_i \leq 1 \) is a fraction between 0 and 1.
   The problem is to determine if you can pack all m objects into the k bins you are given.
   Define a verifier that proves that the bin packing problem is in NP.
   Say what the input to the verifier is, what the verifier does on its input, and why the verifier runs in polynomial time.

3. i. Show that the intersection of two NP languages K and L is also in NP.
   ii. Prove that if a language K is P-time map-reducible to a language L and L is in NP then K is in NP.
   iii. Give an example where the converse of ii. fails. Specifically, an example where a language K is P-time map-reducible to a language L and K is in NP but L is not in NP.

4. In a graph G, an independent set is a collection of vertices C where no two vertices in C have an edge in G between them.
   The IS (independent set) problem is: Given a graph G and an integer k, does G have an independent set of k or more vertices.
   i. (3 points) Give an example of a connected graph with 8 vertices which contains an independent set of size 5 and a clique of size 4.
   ii. (7 points) Find a reduction from the clique problem (given in the textbook) to the IS problem. Explain what the reduction should do, define the reduction, and explain why it does what it is supposed to.
5. i. Page 326, problem 7.35 (7 points)
   (Note: Here, for dominating sets, you should assume that any vertex is adjacent to itself. So
   D is a dominating set for G means that, for any vertex v of G is either in D or is adjacent to
   some vertex in D.)
   ii. You should also show that the dominating set problem is in NP here. (3 points)

Part 2: These problems are good practice and you should try them. They will not be graded

1. Give an example of a bin packing algorithm which always finds a packing which uses no more
   than twice the optimal number of bins in its packings.
   (This is called a 2-approximation algorithm for bin packing.)

2. Show that the union of two NP languages is in NP.

3. Do you think that NP is closed under complement? That is, if a language L is in NP then
   must its complement also be in NP?

4. Page 323, problem 7.9

5. Page 323, problem 7.18