1. i. Show that $P$ is closed under intersection.
   (This means that if languages $J$ and $L$ are both in $P$, then so is $J \cap L$.)
   
   ii. Show that $JL$ (the concatenation of $J$ and $L$) is in $P$.
   (Here $J$ and $L$ are in $P$ as in part (i), and $JL = \{ xy \mid x \in J \text{ and } y \in L \}$.)
   Here you should calculate the complexity of your $P$-time algorithm for $JL$.
   
   iii. Show that $NP$ is closed under union.

2. The Graph accessibility Problem (GAP) = \{(G,s,t) \mid G \text{ is a directed graph where } s \text{ and } t \text{ are two vertices in } G \text{ and there exists a path from } s \text{ to } t \text{ in } G\}.

   Show that GAP is in $P$. You need not use a TM just give a pseudo-code algorithm. You should do this using an algorithm of your choosing but need to do a detailed analysis of your algorithm to show its polynomial time bound.

3. Note: Some of the definitions and ideas for this problem can be found on page 299 of our textbook.
   
   i. Give an example of a Boolean formula $F$ which contains at least 2 different variables, and where $F$ is satisfiable and its negation ($\neg F$) is not satisfiable.
   
   ii. Give an example of a Boolean formula $F$ which contains at least 3 different variables, and where $F$ is satisfiable and has exactly 5 satisfying truth assignments.

4. Show that the Hamiltonian path problem (which was discussed in lab section) is in $NP$.

5. Page 323, problem 7.12

6. (Just so you won’t forget about the first 1/2 of the class. No partial credit, no need to justify your answer.)

   Say whether each of i. - v. is True or False.
   
   i. Any language $J$ with $\text{HALT} - J$ finite is undecidable. ($\text{HALT} = \text{the halting problem}$.)
   
   ii. If $A$ is recognizable and $B \subseteq A$ then $B$ is recognizable.
iii. The disjoint union of a finite set and an infinite set is always infinite.
iv. If $J$ is reducible to $K$ and $K$ is recognizable then $J$ is also recognizable.
v. If $L$ is an undecidable problem, then any language $J$ where $J - L$ is finite is also undecidable.