At the top of your homework paper write your name, the class number (CS 332) and also the homework assignment number (HW 5), and write the number of the problem at the top of your problem solution for each problem.

Reading: Chapter 7: Pages 292-311.

PROBLEMS:

1. i. Show that P is closed under union.
   (This means that if languages L1 and L2 are both in P, then so is L1 ∩ L2.)
ii. Show that JL (the concatenation of J and L) is in P.
   (Here JL = \{ xy | x ∈ J and y ∈ L \}.)
   Here you should calculate the complexity of your P-time algorithm for JL.
iii. Show that NP is closed under union.

2. The Graph accessibility Problem (GAP) = \{ (G,s,t) | G is a directed graph where s and t are two vertices in G and there exists a path from s to t in G \}.
   Show that gap is in P. You need not use a TM just give a pseudo-code algorithm. You should do this using an algorithm of your choosing but need to do a detailed analysis of your algorithm to show its polynomial time bound.

3. Note: Some of the definitions and ideas for this problem can be found on page 299 of our textbook.
   i. Give an example of a Boolean formula F which contains at least 2 different variables, and where F is specifiable and its negation (not)F is not specifiable.
   ii. Give an example of a Boolean formula F which contains at least 3 different variables, and where F is satisfiable and has exactly 4 satisfying truth assignments.

4. Show that the Hamiltonian path problem (which was discussed in lab section) is in NP.

5. Page 323, problem 7.12

6. (Just so you won’t forget about the first 1/2 of the class. No partial credit, no need to justify your answer.)
   Say whether each of i. - v. is True or False.
   i. Any language J with HALT - J finite is undecidable. (HALT = the halting problem.)
   ii. If A is recognizable and B ⊆ A then B is recognizable.
iii. The disjoint union of a finite set and an infinite set is always infinite.
iv. If $J$ is reducible to $K$ and $K$ is recognizable then $J$ is also recognizable.
v. If $L$ is an undecidable problem, then any language $J$ where $J - L$ is finite is also undecidable.