CS 332 - Homework 6 - brief answers

Due: Thursday, May 1

Reading: Chapter 7: Pages 311-120, Chapter 8, pages 331-341, and pages 348-354.

PROBLEMS:

1. i. If it turns out that P=NP, then which of the following languages are in P?
   (briefly explain your answer for any 2 of the following 4 parts a,b,c,d.)
   a. The independent set problem.
   b. The problem which takes as input a natural number n and print out all of the permutations of 1,2,3,...,n.
   c. The complement of the Hamiltonian path problem.
   d. The halting problem.
   Answers: a and c are in P. b and d are not.
   a and c are in P as the IS problem is in NP, hence in P as we are assuming P=NP for this problem. The HP problem is in P and then so is its complement.
   b is not in NP as it is not a decision problem.
   d is not in NP as the halting problem is undecidable and P/NP problem does not pertain here.

2. The double-SAT problem = \{ F \mid F\text{ has at least 2 satisfying assignments}\}
   i. Explain why the identity function on formulas, that is the function id where id(F)=F, is NOT a correct reduction of SAT to double-SAT.
   Answer: The identity function id does not work because x is in SAT but x is not in double SAT even though id(x) = x.
   ii. Prove that the SAT problem is polynomial reducible to double-SAT problem.
   Answer: A reduction f which reduces SAT to double-SAT is f(F) = F \lor (r \land \neg r) where r is a variable that does not appear in F.
   Note that the formula f(F) has no satisfying assignments if if F has none, 2 satisfying assigns if F has 1, and more than 2 if F has 2 or more. Hence it is a correct reduction.

3. i. Give an example of a graph G which contains both a clique of size 4 and an independent set of size 3.
   Answer: Not graded - this problem was left in by accident.
The 3 Color Problem is the set of all graphs G whose nodes can be colored using only 3 colors and where any two connected nodes have different colors.

ii. Give an example of a graph with no 4-clique in it which cannot be colored by 3 colors. (Note: A 4-clique is not 3-colorable.)
Answer: Too hard to draw but just 6 triangles drawn connected from left to right and then one more edge connecting the leftmost point with the rightmost of the graph.

iii. The 3 Color Problem is in NP. What is the certificate which a verifier might use to show this?
Answer: The verifier V takes inputs a graph G and a certificate c which is an assignment of colors to G’s vertices. V(G,c) then checks if c uses only 3 colors (at most) and no edge in G has vertices colored with the same color. If so, V accepts, else it rejects.

The next three problems are for practice for the final, but they are not to be turned in or graded.

4. Consider the statement:
Either prove or disprove this statement: There is a universal Turing machine which runs in polynomial time.

5. Which one of the following 4 statements is NOT true about a universal Turing machine (UTM).
   a. The set of inputs for which the UTM halts is recognizable.
   b. You could actually write the program of such a machine.
   c. A UTM can be used to decide an unsolvable problem.
   d. The UTM can simulate the computation of any Turing machine on any input.

6. a. T or F (justify your answer)
   If a language L is in Time \( n^3 \) then L is in Space \( n^6 \).
   b. T or F (justify your answer)
   If a language L is in Space \( n^3 \) then L is in Space \( n^6 \).