CS 332 - Homework 6 - last one (first draft)

Due: Thursday, May 1

Reading: Chapter 7: Pages 311-120, Chapter 8, pages 331-341, and pages 348-354.

PROBLEMS:

1. i. Give an example of a Boolean formula F which contains at least 2 different variables, and where F is satisfiable and its negation (not)F is not satisfiable.
   
   ii. Give an example of a Boolean formula F which contains at least 3 different variables, and where F is satisfiable and has exactly 5 satisfying truth assignments.

2. i. If it turns out that P=NP, then which of the following languages are in P?
   (briefly explain your answer for any 2 of the following 4 parts a,b,c,d.)
   
   a. The independent set problem.
   
   b. The problem which takes as input a natural number n and print out all of the permutation of 1,2,3,...,n.
   
   c. The complement of the Hamiltonian path problem.
   
   d. The halting problem.

3. The double-SAT problem = \{F | F has at least 2 satisfying assignments\}
   
   i. Explain why the identity function on formulas, that is the function id where id(F)=F, is NOT a correct reduction of SAT to double-SAT.
   
   ii. Prove that the double-SAT problem is polynomial reducible to the SAT problem.
   (That is, you need to define a function g:{Boolean formulas} \rightarrow {Boolean formulas} which does correctly reduce SAT to double-SAT.

4. i. Give an example of a graph G which contains both a clique of size 4 and an independent set of size 3.
   
   The 3 Color Problem is the set of all graphs G whose nodes can be colored using only 3 colors and where any two connected nodes have different colors.
   
   ii. Give an example of a graph with no 4-clique in it which cannot be colored by 3 colors.
   (Note: A 4-clique is not 3-colorable.)
   
   iii. The 3 Color Problem is in NP. What is the certificate which a verifier might use to show this?

The next three problems are for practice for the final, but they are not to be turned in or graded.
5. Consider the statement:
   Either prove or disprove this statement: There is a universal Turing machine which runs in polynomial time.

6. Which one of the following 4 statements is NOT true about a universal Turing machine (UTM).
   a. The set of inputs for which the UTM halts is recognizable.
   b. You could actually write the program of such a machine.
   c. A UTM can be used to decide an unsolvable problem.
   d. The UTM can simulate the computation of any Turing machine on any input.

7. a. T or F (justify your answer)
   If a language $L$ is in Time ($n^3$) then $L$ is in Space ($n^6$).
   b. T or F (justify your answer)
   If a language $L$ is in Space ($n^3$) then $L$ is in Space ($n^6$).