CS 332 - Homework 6 - Last One

Due: Tuesdays, May 1

Reading: None

PROBLEMS

Part 1: 4 of the following 5 problems will be graded. You can choose which one to leave out. There are 40 points total. (10 points each).

1. Consider the independent set (IS) problem which we discussed in class and appeared in HW 5.
   Show that if P=NP then there in algorithm A running in polynomial time which takes as input a graph G and which outputs a largest independent set I in G. (Largest means having the most pairwise independent vertices possible, and it may not be unique.)
   Hint: Take a look at the solution to problem 7.40 on page 327 in the text.

2. You may assume that A and B below are both sets of Boolean strings.
   (i). Prove that SPACE \( n^3 \) is closed under difference. That is, if A and B are both in SPACE \( n^3 \) then so is A-B.
   (ii). Prove that SPACE \( n^3 \) is closed under complement. That is, if A is in SPACE \( n^3 \) then so is the complement of A. You can use part (i) of this problem if you like.

3. Explain briefly why the clique problem is in PSPACE.
   What I am asking for is a description of how you can solve an instance of the clique problem using only an amount of space (memory) which is some polynomial in the size of the instance.

4. Prove that NP is a subset of PSPACE.

5. The definition of PSPACE-complete is very similar to NP-complete. It can be found on page 337 of the textbook. And an example of such a problem can be found on page 339.
   Prove that if C is a PSPACE-complete set and C\(\in\)NP then NP=PSPACE.

Part 2: These problems are good practice and you should try them. They will not be graded.

1. Give an example of a bin packing algorithm which always finds a packing which uses no more than twice the optimal number of bins in its packings.
   (This is called a 2-approximation algorithm for bin packing.)
   Maybe you can show that the first fit bin packing algorithm gives a 2-approximation?

2. Give an example of a propositional formula with exactly three variables which is true for every possible truth assignment. (Such a formula is called a tautology.)
3. Show that the intersection of a P problem and an NP problem is in NP.

4. Do you think that NP is closed under complement? That is, if a language L is in NP then its complement must also be in NP?

   I’m not looking for a proof here just some intuition about the problem, for example explaining why the “usual” reason that sets like P are closed under complement does not work for NP.

5. Page 323, problem 7.12