Some background questions to go over in class. Your participation in class is needed!

1. i. Given the sets $A = \{1, 3, 5\}$ and $B = \{2, 3, 4, 5, 6\}$, what is $A \cap B$?
What is $A - B$? Is $A \subseteq B$?
Name 2 elements of $A$. Name 2 subsets of $B$.
What is $A \times B$ (the set of ordered pairs from $A$ and $B$)?
Let $|A|$ be the size of a set $A$. What is $|A \times B|$?

ii. Answer the same questions when $A = \{n \mid n \text{ is an even natural number}\}$ and $B = \{z/2 \mid z \text{ is a natural number}\}$.

2. In this class we use $\exists$ to mean "there exists" and $\forall$ to mean "for all". So for example,

$\exists x \forall y \ x + y = y$ means "The exists an $x$ such that for all $y$, $x + y = y$".

This is a true statement about the integers because $x = 0$ makes the statement true.

However it is not true about the positive integers.

Questions:

i. Consider the statement $\forall x \forall y$ (if $x$ and $y$ are odd then $xy$ is odd).

Is this true or false? Why?

ii. Consider statement $S = \forall x \forall y \exists z \ x + z = y$.

Is $S$ true about the natural numbers? the integers? the rational numbers? the real numbers? the numbers $1, 2, 3, \ldots, p \pmod{p}$, $p$ is prime? the numbers $1, 2, 3, \ldots, p \pmod{p}$, $p$ is not prime?

3. A function $f$ is "one to one" or 1-1 or injective if for any 2 different inputs $x$ and $y$, $f(x)$ is not the same as $f(y)$.

(i). Is $f(x) = 2x$ one to one? How about $f(x) = (1/2)x$? How about $f(x) = x^2$?

(ii). Using quantifiers write out the statement that says that a function $g$ is 1-1.

A function $f$ is "onto" or surjective if for any possible value $z$ of $f$ there is an input $x$ for $f$ such that $f(x) = z$.

(iii). Is $f(x) = 2x$ onto? How about $f(y) = y^2$? How about $f(n) = n+1$?
4. i. Let $A = \begin{pmatrix} 4 & 6 & 5 & 2 \\ -4 & 2 & 3 & 1 \\ 7 & 0 & 8 & 1 \end{pmatrix}$ and let $B = \begin{pmatrix} 4 & 6 \\ 3 & -2 \\ -4 & 7 \\ 1 & 8 \end{pmatrix}$.

What is $AB$? How many multiplications and additions does it take to multiply an $m \times n$ matrix by an $n \times o$ matrix?

ii. Consider the statement $\exists B \forall A \ AB = A$ for matrices $A$ and $B$. Is this statement true or false? Why?

5. Find a simple closed-form solution for the following sums and prove the closed-form solution is correct using induction.

i. $$\sum_{i=0}^{n} 3i = 3 + 6 + 9 + 12 + \ldots + 3n$$

ii. $$\sum_{i=0}^{n} 4^i = 1 + 4 + 4^2 + 4^3 + \ldots + 4^n$$

For a good on-line exposition of some of these basic facts see

www3.cs.stonybrook.edu/~pfodor/courses/CSE215/L09_Induction.pdf