CS 530 - Fall 2017

CS 530 - First Day(s) Class Review

Some background questions to go over in class. Your participation in class is needed!

1. i. Given the sets A = {1, 3, 5 } and B= {2,3,4,5,6}, what is A $\cap$ B ?
What is A $-$ B ? Is A $\subseteq$ B ?
Name 2 elements of A. Name 2 subsets of B.
What is A $\times$ B (= the set of ordered pairs from A and B) ?
Let $|A|$ be the size of a set A. What is $|A \times B|$?

ii. Answer the same questions when A = {n | n is an even natural number} and B= {z/2 | z is a natural number }.

2. In this class we use $\exists$ to mean "there exists" and $\forall$ to mean "for all". So for example,

$\exists$x $\forall$y x+y = y means "The exists an x such that for all y, x+y=y".

This is a true statement about the integers because x = 0 makes the statement true.

However it is not true about the positive integers.

Questions:
i. Consider the statement $\forall$x$\forall$y (if x and y are odd then xy is odd).

Is this true or false ? Why?

ii. Consider statement S =$\forall$x$\forall$y$\exists$z x+z=y.

Is S true about the natural numbers ? the integers ? the rational numbers ? the real numbers ? the numbers 1,2,3,...,p (mod p), p is prime ? the numbers 1,2,3,...,p (mod p), p is not prime ?

3. A function f is "one to one" or 1-1 or injective if for any 2 different inputs x and y, f(x) is not the same as f(y).

(i). Is f(x) = 2x one to one ? How about f(x) = (1/2)x ? How about f(x) = x^2 ?

(ii). Using quantifiers write out the statement that says that a function g is 1-1.

A function f is "onto" or surjective if for any possible value z of f there is an input x for f such that f(x) =z.

(iii). Is f(x) = 2x onto ? How about f(y) = y^2 ? How about f(n) = n+1 ?

4. i. Let $A = \begin{pmatrix} 4 & 6 & 5 & 2 \\ -4 & 2 & 3 & 1 \\ 7 & 0 & 8 & 1 \end{pmatrix}$ and let $B = \begin{pmatrix} 4 & 6 \\ 3 & -2 \\ -4 & 7 \\ 1 & 8 \end{pmatrix}$

What is AB? How many multiplications and additions does it take to multiply an $m \times n$ matrix by an $n \times o$ matrix?

ii. Consider the statement $\exists B \forall A \ AB = A$ for matrices A and B. Is this statement true or false? Why?

5. Find a simple closed-form solution for the following sums and prove the closed-form solution is correct using induction.

i.

$$\sum_{i=0}^{n} 3i = 3 + 6 + 9 + 12 + ... + 3n$$

ii.

$$\sum_{i=0}^{n} 4^i = 1 + 4 + 4^2 + 4^3 + ... + 4^n$$

For a good on-line exposition of some of these basic facts see

www3.cs.stonybrook.edu// pfodor/courses/CSE215/L09$_1$ – *Induction.pdf*