Date Due: Tuesday, September 8, in class
Reading: Chapters 1-5, look over and read whatever seems new to you.

Also, please read the handout on introductory randomized algorithms which can be found from the class homepage right after the pointer to HW 0.

This homework is just a check to make sure people have some necessary math background. You need to get each question “mostly right” before you do HW 1. You can do them multiple times if needed. This quiz will not be graded and is not part of the final grade.

1. (i). Give a precise definition of $O(n^3)$.

$O(n^3)$ is a set of functions $f: \mathbb{N} \to \mathbb{R}^+$. Specifically it is all functions $f$ for which there exists a natural number $m$ and a constant $c>0$, such that for all $n > m$, $f(n) < cn^3$.

(ii). Give an example of a function $g(n)$ which is not in $O(n^3)$.

Using your definition given in (i) prove that this is the case for $g(n)$.

We will show that $g(n) = n^3 \log n$ is not in $O(n^3)$.
To do this we need to show that for any chosen $m$ and $c$ as above, there is a natural number $n>m$ with the property that $(n^3 \log n) > cn^3$.
Recall that $\log n$ is unbounded and let $n>m$ be such that $\log n > c$. It then follows that $(\log n n^3) > cn^3$.

2. Define $h(t) = 2$ if $t=1$ and $h(t)= t+h((t-1))$ if $t$ is any integer greater than 1.

(i). What is $h(5)$ ? $h(5) = 16$

(ii). What is $h(t)$ ? That is find a closed form which expresses $h(t)$ in terms of $t$ but not using any earlier values of $h$.

$h(t) = t(t+1)/2+1$

3. Consider the star problem as discussed in class, but rather than 3 people there are 7 who are given stars. The object of the problem is the same.

Find a solution to the problem with is correct with probability 7/8.

Answer to appear soon - based on ideas of a length 7 binary error correcting code.

4. Let $A$ be the 3 by 3 matrix

\[
\begin{pmatrix}
-1 & 0 & 4 \\
\end{pmatrix}
\]
3 6 2
1 0 5
i. What is the determinant of A? \( \text{det}(A) = -54 \).

ii. What is the permanent of A? \( \text{perm}(A) = -6 \)

iii. What is the inverse of A?
\[
A^{-1} =
\begin{pmatrix}
-5/9 & 0 & 4/9 \\
13/54 & 1/6 & -14/54 \\
1/9 & 0 & 1/9
\end{pmatrix}
\]

iv. Draw a graph G which has matrix A as its adjacency matrix. (In this case we allow G to have edges from a vertex to itself.)

5. Let \( g(x) = x^3 - 3x^2 + 5 \).

i. What is \( g(x) + g(x) \)? \( 2x^3 - 6x^2 + 10 \)

ii. What is \( g(x) \times g(x) \)? \( x^6 - 6x^5 + 10x^3 + 9x^4 - 30x^2 + 25 \)

iii. Let \( f(x) = 4x^3 - 10 \). What function \( h(y) \) is the inverse of \( f \), that is \( f^{-1} \).
\( h(y) = \) the cube root of \( (y+10)/4 \).