CS 530: Algorithms --- Steve Homer--- Fall 2017

Homework 1 --- Due Tuesday, September 19

Reading: 1. Chapter 26, pages 708-736

2. Chapters 1-4, look over and read whatever seems new to you.

Problems:

1. (12 points). Briefly explain your reasoning in (i) - (iv).
   (i) Is $2^{n+2}$ in $O(2^n)$?
   (ii) Is $2^{2n}$ in $O(2^n)$?
   (iii) Is $(2^n)^2$ in $O(2^n)$?
   (iv) Is $n!$ in $O(2^n)$?

2. (6 points) Page 62, #3-4 parts e, and g. Justify your answers here.

3. Use the Ford Fulkerson algorithm to find the the maximum flow in the following flow graph.
   Find the min s-t cut of the graph as well.

4. (5 points) Suppose you are given a flow graph $G$ and also given a max flow $f_{max}$ for $G$.
   Show that you can then find a sequence of at most $|E|$ augmenting paths which will result in the flow $f_{max}$.

5. (10 points)
   (i). Recall the min cut problem for undirected graphs. Give an example of a graph $G$
   whose is min cut is not any single vertex of the graph. That is, for any vertex $v$ in $G$ the
   cut consisting of $\{v\}$ on one side and all other vertices on the other is not a min cut.

   (ii). Assume $G$ is a 10 node graph and that $G$ has a min cut of cost 5. Explain why $G$ must
   have at least 25 edges.
(iii). Assume $H$ is a 10 node graph which is a simple cycle. That is $H$ is connected and every node in $H$ has exactly 2 neighbors. Clearly the size of $H$'s min cut is 2. How many different min cuts does $H$ have? Explain your reasoning.

6. (10 points) A **centroid** of a tree with $n$ vertices is a vertex such that its deletion leaves no subtree with more than $n/2$ vertices.

Is the centroid of a graph always a unique vertex? Explain.

Write an algorithm to find a centroid of a tree.

Show how your algorithm works on the graph below. What is the complexity of your algorithm?

You should be able to get a linear solution although partial credit will be given for a nonlinear algorithm.

Note: you are not required to prove that your algorithm is correct.

Picture: Vertex $c$ is the centroid of graph $G$. It is unique in this case.