CS 530 - Fall 2018

CS 530 - First Days Class Review

Reading: Skim through the first 4 chapters of the book. If you see some topic you don't recall or understand, spend a few minutes reviewing it. If necessary you can find other sources to read, for example in the appendices of our text or in some other algorithms book or on-line.
Read the first two sections of Chapter 28 in the text on matrix operations.

Below are some background questions to go over in class. Your participation in class is needed!

1. i. Given the sets A = {1, 4, 7 } and B= {2,3,4,5,6}, what is A $\cap$ B ?
What is A $-$ B ? A $\cap$ B ? Is A $\subseteq$ B ?
Name 2 elements of A. Name 2 subsets of B.
What is A $\times$ B (= the set ordered pairs from A and B) ?
Let $|A|$ be the size of a set A. What is $|A \times B|$?

ii. Recall that natural numbers N = {0,1,2,...}, Z^+ = {positive integers}, and Z = {all integers}.
Answer the same questions as in i. when A = {n | n is an odd natural number} and B= {z/2 | z is a natural number }. (So A $\subseteq$ N, while B $\subseteq$ Q = the set of all rational numbers.)

2. In this class we use $\exists$ to mean "there exists" and $\forall$ to mean "for all". So for example,

$\exists$x $\forall$y x+y = y means "The exists an x such that for all y, x+y=y".

This is a true statement about the integers because x = 0 makes the statement true. However it is not true about the positive integers.

i. Consider the statement $\forall$x$\forall$y (if x and y are odd then xy is odd).

Is this true or false about N ? about Z ? Why?

ii. Consider statement S =$\forall$y$\exists$z z+z=y.
Is S true about the natural numbers ? the integers ? Q ? the real numbers ?

3. A function f is "one to one" or 1-1 or injective if for any 2 different inputs x and y, f(x) is not the same as f(y).

(i). Is f(x) = 2x one to one ? How about f(x) = (1/2)x ? How about f(x) = x^2 ?

(ii). Using quantifiers write out the statement that says that a function g is 1-1.

A function f is "onto" or surjective if for any possible value z of f there is an input x for f such that f(x) =z.

(iii). Is f(x) = 2x onto ? How about f(y) = y^2 ? How about f(n) = n+1 ?

4. i. Let A = $\begin{matrix} 4 & 6 & 5 & 2 \\ -4 & 2 & 3 & 1 \\ 2 & 0 & -1 & 1 \end{matrix}$ and let B = $\begin{matrix} 4 & 6 \\ 3 & -2 \\ -4 & 7 \\ 0 & 1 \end{matrix}$

What is the dimension of AB? How many multiplications and additions does it take to multiply an m×n matrix by an n×r matrix?

ii. Show how to use block matrix multiplication in order to break up the computation of AB, a 3 by 2 matrix, into 2 multiplications of 3 by 2 (A) and 2 by 2 matrices (B).

iii. Consider the statement True or false ? For matrices A and B, $\exists B \forall A \; AB = A$. Why or why not?
How about $\exists A \forall B \; AB = A$?

5. Find a simple closed-form solution for the following sums and prove the closed-form solution is correct using induction.

i.

$$\sum_{i=0}^{n} 3i = 3 + 6 + 9 + 12 + ... + 3n$$

.

ii.

$$\sum_{i=0}^{n} 4^i = 1 + 4 + 4^2 + 4^3 + ... + 4^n$$

6. (i). Put in increasing order: $O(n^{log\ n}), O(n\ log\ n), O(1.1^n), O(2^{log\ n}), O((log\ n)^2)$.

(ii). Explain why the first function class in your list is less than the second. Be precise.

(iii). Prove that (1) $n^2 log\ n$ is not in O($n(logn)^2$).
By prove I mean state precisely what the two statements mean and then use the precise statement in your arguments/proofs that the statements are true.

7. (a) Describe (write) an algorithm which takes an undirected graph G and two vertices v1 and v2 in that graph and decides if there is a path from v1 to v2 in G.
Your algorithm should be given in pseudo-code, in a manner similar to how algorithms are presented in our textbook.
(b) Show how your algorithm works on a graph with about 7 nodes.
(c) Is your algorithm efficient ?
(Here you need not answer formally, just say if you think it is an efficient algorithm or not and say why. No proof needed.)