Quiz 2 Answers

FALL 2018
(Green/Blue)
(White)

#1 (Multiple choice)

i) A is False: \[ \triangle \] prob. of finding mincut = 1
   B is True: \[ \text{Prob(finding mincut)} \geq \frac{2}{5} = \frac{1}{15} > \frac{1}{25} \]
   C is False; counter example: complete graph on 5 nodes

ii) A. Outputs KL \neq M
    B. Output is correct since KL \neq M.
    C. Error probability is 2/8, since algo: outputs KL=M iff V=(0,0,0)^T or (0,0,1)^T

iii) A. True; \text{deg}(g) \neq \text{deg}(f)
    B. False; \text{deg}(g) is unknown. \text{deg}(f-g) could be arbitrary
    C. False; ditto as above

2. i) Flip c 3 times to get result r.
   If r is in {HHH, HHT, HTT} let \( b = H \),
   If r is in {HHT, HTT, THT, TTH, TTT} let \( b = T \),
   \[ \text{Prob}(b=H) = \frac{3}{8} \text{ and } \text{Prob}(b=T) = \frac{5}{8} \text{ as expected} \]
   Sequence of coin flip has prob = \( \frac{1}{8} \).

ii) Flip c 4 times to get result r. Consider r as a 4 bit +
   integer from 1 to 16,
   If \( r \in \{1, 2, \ldots, 8\} \) then reflip. (Try again)
   If \( r \in \{9, 10, 11, 12, \ldots, 16\} \) let \( b = H \),
   If \( r \in \{10, 11, 12, \ldots, 16\} \) let \( b = T \)
   Note: \text{prob (you have to reflip forever)} = 0. (Getting The values of \( r \) you obtain, one of \( H, T \) values are all equally likely, and \text{prob}(b=H) = \frac{3}{16}, \text{prob}(b=T) = \frac{13}{16}.}
(i) REDRAWN $G = n$ max-cut has size $\leq 7$, 
(edges marked by x's) 
\[ \text{max cut} = (a, c, e, f) (b, d, e) \] 

(ii) 
\[ G_n = (V, E) \text{ where } V = \{1, 2, 3, \ldots, n\} \]
\[ E = \{ (1, 2), (2, 3), (3, 4), \ldots, (n-1, n), (n, 1) \} \]

The max-cuts of $G_n$ have size 2. 
Any 2 edges determine a min-cut. There are $n$ edges in $G_n$, so $\frac{n(n-1)}{2}$ pairs of edges, and so $\frac{n(n-1)}{2}$ min-cuts.

# 4. (Farms)
- \( F_1 = P_2^k \) if and only if \( P_{F_1}(\alpha) \equiv P_{F_2}^k(\alpha) \iff P_{F_1}(\alpha) \equiv P_{F_2}(\alpha^k) \)
- Degree of \( P_{F_2}^k(\alpha^k) = d_2 k \). Let \( d \triangleq \max d, d_2 k \).

Algorithm
\[ \text{i/p: } P_{F_1}(\alpha), P_{F_2}(\alpha) \]
1. Let \( \mathcal{S} \) be a set of size $8d+1$.
2. Pick \( \gamma \in \mathcal{S} \) uniformly at random.
3. Output \( F_1 = F_2^k \) if \( P_{F_1}(\gamma) = P_{F_2}(\gamma^k) \).
4. Output \( F_1 \neq F_2^k \) otherwise.

Analysis
if \( P_{F_1}(\alpha) \equiv P_{F_2}(\alpha^k) \) the algo. will always output the correct answer. Otherwise, the probability that the algo. makes an error
\[ \leq \frac{d}{15} \leq \frac{1}{8} \]
# QUIZ 2 ANSWERS

Q5 530
FALL 2018
BLUE/GREEN

1. A. True; \( \deg(g) \neq \deg(f) \)
B. False; \( \deg(g) \) is unknown. \( \deg(f-g) \) could be arbitrary.
C. False; ditto

2. A. Algo. outputs \( KL \neq M \)
B. Correct; since \( KL \) is in fact not equal to \( M \).
C. \( \frac{1}{8} \)

3. A. False; Counterexample: \( G_n \)
B. False; Counterexample - Consider a graph on \( n \) vertices where \( n-1 \) vertices form a cycle. There is an edge from one of the cycle-vertices to the \( n \)th vertex. Prob. of finding mincut here is \( \frac{2}{n} \). Statement false for \( G_{151} \).
C. False; \( \square \) is counter example.

2(i) Flip c 3 times to get result \( r \)
If \( r \) is in \( \{HHH, HHT, HTT, TTH\} \) let \( b = H \),
If \( r \) is in \( \{HHT, THT, TTHT, TTT\} \) let \( b = T \).
Prob \( (b=H) = \frac{3}{8} \) and Prob \( (b=T) = \frac{5}{8} \) as each sequence of coin flip has prob \( = \frac{1}{8} \).

2(ii) Flip c 4 times to get result \( r \). Consider \( r \) as a 4 bit integer from \( 1 \) to \( 16 \).
If \( r \in \{1, 2, \ldots, 63\} \) then reflip. (Try again)
If \( r \in \{7, 8, 9, 98\} \) let \( b = H \), If \( r \in \{10, 11, 12, \ldots, 15\} \) let \( b = T \).
Note: prob \( \text{(you have to reflip forever)} = 0 \). Getting the values of \( r \) you obtain, one of \( 10 \) values are all equally likely, and prob \( (b=H) = \frac{3}{16} \), prob \( (b=T) = \frac{7}{16} \).
3. RE-DRAWING $G = \begin{array}{c}
\begin{array}{c}
\text{max-cut has size 7,}
\text{edges marked by x's)}
\maxcut = (a, c, d) (b, d, e)
\end{array}
\end{array}
\begin{array}{c}
(\exists i) \quad G_{n,i} = (V, E) \text{ where } V = \{1, 2, 3, \ldots, n\},
E = \{(1, 2), (2, 3), (3, 4), \ldots, (n-1, n), (n, 1)\}
\text{The min-cuts of } G_{n,i} \text{ have size 2.}
\text{Any 2 edges determine a min-cut. There are n edges in } G_{n,i}, \text{ so } \frac{n(n-1)}{2} \text{ pairs of edges, and so } \frac{n(n-1)}{2} \text{ min-cuts.}
\end{array}

\#4. (Games)
- $F_1 = F_2^k$ if and only if $P_{F_1}(x) \equiv P_{F_2^k}(x) \iff P_{F_1}(x) \equiv P_{F_2}(x^k)$.
- Degree of $P_{F_2}(x^k) = d_2 k$. Let $d \equiv \max d_i, d_2 k$.

\underline{Algorithm}
\begin{enumerate}
\item \text{Safek be a set of size } 8d+1.
\item \text{Pick } \gamma \in S \text{ uniformly at random.}
\item \text{Output } F_1 = F_2^k \text{ if } P_{F_1}(\gamma) = P_{F_2}(\gamma^k).
\item \text{Output } F_1 \neq F_2^k \text{ otherwise.}
\end{enumerate}

\underline{Analysis}
\begin{enumerate}
\item \text{If } P_{F_1}(x) \equiv P_{F_2}(x^k), \text{ the algo. will always output the correct answer. Otherwise, the probability that the algo. makes an error } \leq \frac{d}{15} \leq \frac{1}{8}.
\end{enumerate}