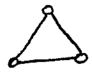
CS 530
FALL 2018
~~(GREEN/BLUE)~~
(WHITE)

QUIZ 2 ANSWERS

#1 (MULTIPLE CHOICE)

(i) A is False;  , prob. of finding mincut = 1

B. is True; Prob(finding mincut) $\geq \frac{2}{6 \cdot 5} = \frac{1}{15} \geq \frac{1}{25}$.

C is False; Counter example: complete graph on 5 nodes.

(ii) A. Outputs $KL \neq M$

B. Output is correct since $KL \neq M$.

C. Error probability is 2/8, since algo outputs $KL = M$ iff $V = (0,0,0)^T$ or $(0,0,1)^T$.

(iii). A. True ; $\deg(g) \neq \deg(f)$

B. False ; $\deg(g)$ is unknown. $\deg(f-g)$ could be arbitrary.

C. False ; ———— ditto as above ————

2 (i) Flip c 3 times to get result r
If r is in {HHH, HHT, HTH} let b = H,
If r is in {HTT, THH, THT, TTH, TTT} let b = T,
Prob(b = H) = 3/8 and Prob(b = T) = 5/8 as each
sequence of coin flip has prob = 1/8.

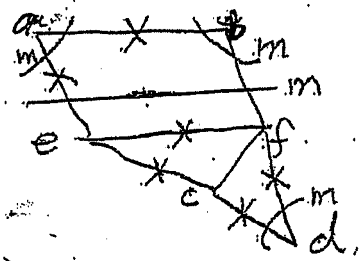
(ii). Flip c 4 times to get result r. Consider r as a 4 bit integer from 1 to 16.

If $r \in \{1, 2, \ldots 6\}$ then reflip. (Try again)

If $r \in \{7, 8, 9\}$ let b = H, If $r \in \{10, 11, 12, \ldots 16\}$ let b = T.

Note: prob (you have to reflip forever) = 0. Getting the values of r you obtain, one of 10 values, are all equally likely, and prob(b=H) = 3/10, prob(b=T) = 7/10.

3 (i) REDRAWN G =



m = $\underline{4}$ min cuts of G all have size $\underline{2}$

max-cut ~~has~~ $\underline{\text{size}=7}$, (edges marked by x's)

maxcut = $\underbrace{(a, c, f)}_{S}\ \underbrace{(b, d, e)}_{T}$

(ii)

$G_n = (V, E)$ where $V = \{1, 2, 3, \ldots, n\}$

$E = \{(1,2)\ (2,3),\ (3,4)\ \ldots\ (n-1, n)\ (n, 1)\}$

The min-cuts of G_n have size 2.

Any 2 edges determine a min-cut. There are n edges in G_n, so $\frac{n(n-1)}{2}$ pairs of edges, and so $\frac{n(n-1)}{2}$ min-cuts.

#4 (Farms)

- $F_1 = F_2^k$ if and only if $P_{F_1}(x) \equiv P_{F_2^k}(x) \Leftrightarrow P_{F_1}(x) \equiv P_{F_2}(x^k)$
- Degree of $P_{F_2}(x^k) = d_2 k$. Let $d \triangleq \max\{d_1, d_2 k\}$

Algorithm

i/p : $P_{F_1}(x)$, $P_{F_2}(x)$

1. Let $S \subseteq F$ be a set of size $8d+1$.
2. Pick $r \in S$ uniformly at random.
3. Output $F_1 = F_2^k$ if $P_{F_1}(r) = P_{F_2}(r^k)$.
4. Output $F_1 \neq F_2^k$ otherwise.

Analysis

if $P_{F_1}(x) \equiv P_{F_2}(x^k)$, the algo. will always output the correct answer. Otherwise, the probability that the algo. makes an error $\leq \frac{d}{|S|}$

$\leq 1/8$.

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#1 (i) A. True; $\deg(g) \neq \deg(f)$

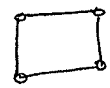
(ii) B. False; $\deg(g)$ is unknown. $\deg(f-g)$ could be arbitrary.

(iii) C. False; ———— ditto ————

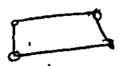
(ii) A. Algo. outputs $KL \neq M$

B. Correct; since KL is infact not equal to M.

C. $1/8$;

(iii) A. False; Counterexample = 

B. False; Counterexample - Consider a graph G_n on n vertices where n-1 vertices form a cycle. There is an edge from one of the cycle-vertices to the n^{th} vertex. Prob. of finding mincut here is $2/n$. Statement false for G_{51}.

C. False;  is counter example.

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(4 bit)
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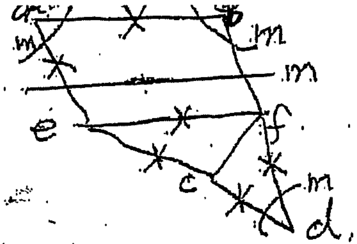

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