Due: Tuesday, December 10 at 11 pm - to be submitted via Gradescope
Note: The last time to turn in any HW will be Wednesday, December 11 at midnight

Reading: Chapter 29, pages 843-885 and the example of the simplex algorithm in the extra reading section of the course home page.

1. Bipartite matching
   (i). Give an example of a connected bipartite graph G which has a maximal matching of some size $t \geq 2$ edges and a maximum matching of size greater than or equal to $t+3$.

   (ii). How many perfect matchings does a complete bipartite graph of 2n nodes (n nodes on each side) have? Give an exact value here (in terms of n). Briefly explain your reasoning.

   (iii). Let $f(n)$ be your answer from part (ii). Show that the number of complete matchings in a complete graph of 2n nodes is strictly bigger than $O(f(n))$. Here the graph is not bipartite. Briefly explain your answer.

2. Carry out the Hopcroft-Karp maximum matching algorithm on the bipartite graph G below. (You can find the algorithm on page 763, it is problem 26-6.)

3. Consider the following linear programming problem;
   maximize $3x + y$

   subject to

   $x + 2y \leq 12$
\[ 2x - 4y \geq -16 \]
\[ 3x + 4y \leq 12 \]

(i) Put the problem into standard LP form. (Don’t forget the non-negativity requirements.)

(ii). Put your answer for part (i) into slack form, and draw the picture of the feasible space for this problem. Label your picture clearly indicating the feasible region, various constraints, and the extreme points.

Say what the extreme points of your drawing are.

(iii). Now do one iteration (pivot) of the simplex algorithm on this LP and say which extreme point is the next basic feasible solution.

4. (i). Give an example of an LP in standard form which has feasible solutions but the point (0,0) is not feasible.

(ii). Give an example of an LP in standard form which has infinitely many optimal solutions.

(iii). Give an example of an LP in standard form with 2 variable in its objective function and which has an unbounded feasible region and has a unique optimal solution.

(iv). Give an example of an LP P in standard form with the property that both P and its dual have no feasible solutions.

5. Problem 29-3, parts a. and b. in the textbook, page 895.

6. Let M be an n by n matrix of 0’s and 1’s. We call M zeroable if there is a sequence of row switches and column switches of M which results in all 0’s on the diagonal of M. (Any off-diagonal elements can be either 0 or 1.)

   a. Give an example of an M with n at least 3 which is not zeroable but which has at least one 0 in every row and in every column.

   b. Give an efficient (polynomial number of steps) algorithm to decide if a matrix A is zeroable or not.

   Write your algorithm using pseudocode, not as a full program in some programming language. Run your algorithm on your graph from part a. and explain how your algorithm works and why it works correctly.

   Hint: One option is to use the max matching of a bipartite graph to solve this problem.