Due Wednesday, June 13

Reading: 1. Chapter 35, pages 1106-1128
   2. If needed, review Chapter 34, pages 1048 - 1070

Problems: 10 points each.

1. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine
time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time
and 1 hour of craftman's time.

In a day, the factory has the availability of not more than 42 hours of machine time and 24
hours of craftman's time.

   i. Write down the linear program corresponding to this problem statement.

   ii. What number of rackets and bats must be made if the factory is to work at full capacity?

   iii. If the profit for selling a racket is 12 dollars and that of a bat is 5 dollars, find the maximum
value for the factory profit when working at full capacity.

Answer:

   i. Let $x$ be the number of tennis rackets made and $y$ be the number of cricket bats made. The
linear program corresponding to this problem is: maximize $x + y$

   subject to
   
   $1.5x + 3y \leq 42$
   $3x + y \leq 24$
   $x \geq 0$
   $y \geq 0$

   ii. If the factory is working at full capacity, $1.5x + 3y = 42$ and $3x + y = 24$. First solve for $y$ in
   terms of $x$.
   
   $3x + y = 24$
   
   $y = 24 - 3x$

   Now plug $24 - 3x$ in for $y$ and solve for $x$.
   
   $1.5x + 3(24 - 3x) = 42$
   $1.5x + 72 - 9x = 42$
   $-7.5x = -30$
   $x = 4$

   Since $y = 24 - 3x$ and $x = 4$, $y = 12$. So, when the factory produces 4 tennis rackets and 12 cricket
   bats it is operating at full capacity.

   iii. Let $x$ be the number of tennis rackets made and $y$ be the number of cricket bats made.
The linear program corresponding to this problem is:
maximize $12x + 5y$
subject to
1. $5x + 3y = 42$
2. $3x + y = 24$
3. $x \geq 0$
4. $y \geq 0$

The only solution to this linear program is the solution to ii., $x = 4$ and $y = 12$ which gives a profit of $12(4) + 5(12) = 108$.

2. i. Put the following linear program into standard form.

maximize $18x + 16y - 3z$
subject to
1. $6x - y \geq -12$
2. $-3x + y - 6z \geq -8$
3. $4x + 6y \geq -14$
4. $x \leq 5$
5. $y \leq 5$
6. $x, y, z \geq 0$

Answer:
The only needed change to make the above LP be in standard form is to change all three constraint to being $\leq$'s. To do this multiply them by -1 and you get:

1. $-6x + y \leq 12$
2. $3x - y + 6z \leq 8$
3. $-4x - 6y \leq 14$

ii. Draw a sketch of the feasible region of this LP
Answer:
iii. Now change the standard form problem into its slack form. State which variable in this slack form are basic and which non-basic, and give the basic feasible solution.

Answer:
maximize \(18x + 16y -3z\)

subject to

\[-6x + y + s_1 = 12\]
\[3x - y + 6z + s_2 = 8\]
\[-4x - 6y + s_3 = 14\]
\[x + s_4 = 5\]
\[y + s_5 = 5\]
\[x, y, z, s_1, s_2, s_3 \geq 0\]

Basic feasible solution is:
\[x = y = 0\] and \[s_1 = 12, s_2 = 8, s_3 = 14, s_4 = 5, s_5 = 5\].

3. Consider the LP:

maximize \(4x + y\)

subject to

\[3x + y \leq 9\]
\[x + y \leq 15\]
\[4x - y \leq 6\]
\[x, y \geq 0\]

i. Draw a picture of the feasible region of this problem and label the extreme points of the region.

Find the values of extreme points (corner points) and circle the one with the largest objective value.

Answer:

\[\begin{align*}
\text{maximize } & 4x + y \\
\text{subject to } & 3x + y \leq 9 \\
& x + y \leq 15 \\
& 4x - y \leq 6 \\
& x, y \geq 0 \\
\end{align*}\]

ii. Now write down the LP problem which is dual to this primal problem.

Answer: The dual problem uses variables \(z_1, z_2, z_3\) and is,

minimize \(9z_1 + 15z_2 + 6z_3\)

subject to
\[3x_1 + 1x_2 + 4x_3 \geq 4\]
\[1x_1 + 1x_2 - 1x_3 \geq 1\]
and \(x_1 \geq 0, x_2 \geq 0, x_3 \geq 0\)

4. i. Give an example of an LP in standard form which has an unbounded feasible region and also has an unique maximum objective value.

Answer:
maximize \(y-x\)
subject to
\(y-1/2x \leq 1\)
\(x, y \geq 0\)

ii. Illustrate your example by graphing the feasible region and showing where the maximum value is in your graph.

Answer:
The maximum object value is the point \(x = 0, y=1\).
The feasible region is the region between the \(x\) axis and the line \(y = 1/2x + 1\) in the positive \(x-y\) quadrant.