Due Wednesday, June 13

Reading: 1. Chapter 35, pages 1106-1128
2. If needed, review Chapter 34, pages 1048 - 1070

Problems: 10 points each.

1. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman’s time in its making while a cricket bat takes 3 hour of machine time and 1 hour of craftman’s time.

   In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftman’s time.

   i. Write down the linear program corresponding to this problem statement.

   ii. What number of rackets and bats must be made if the factory is to work at full capacity?

   iii. If the profit for selling a racket is 12 dollars and that of a bat is 5 dollars, find the maximum value for the factory profit when working at full capacity.

2. i. Put the following linear program into standard form.

   maximize $18x + 16y - 3z$

   subject to

   \[
   
   
   \begin{align*}
   6x - y & \geq -12 \\
   -3x + y - 6z & \geq -8 \\
   4x + 6y & \geq -14 \\
x & \leq 5 \\
y & \leq 5 \\
x, y, z & \geq 0
   \end{align*}
   \]

   ii. Draw a sketch of the feasible region of this LP

   iii. Now change the standard form problem into its slack form. State which variable in this slack form are basic and which non-basic, and give the basic feasible solution.
3. Consider the LP:
maximize \( 4x + y \)
subject to
\( 3x + y \leq 15 \)
\( x + y \leq 9 \)
\( -4x + y \leq 6 \)
\( x, y \geq 0 \)

i. Draw a picture of the feasible region of this problem and label the extreme points of the region.

Find the values of extreme points (corner points) and circle the one with the largest objective value.

ii. Now write down the LP problem which is dual to this primal problem.

4. i. Give an example of an LP in standard form which has an unbounded feasible region and also has an unique maximum objective value.

ii. Illustrate your example by graphing the feasible region and showing where the maximum value is in your graph.